A Short Introduction into Fine-Grained and Parameterized Complexity

C.M.F. Swennenhuis

Department of Mathematics and Computer Science, Eindhoven University of Technology
Example of a Bakery

To bake today:
- White bread
- Croissants
- Pie
- French Bread
- Whole-wheat
- Muffins
- Peppernuts

Fine-Grained Parameterized Complexity of Scheduling and Sequencing Problems
Example of a Bakery

To bake today:
- White bread
- Croissants
- Pie
- French Bread
- Whole-wheat
- Muffins
- Peppernuts

Time to bake: 8 hours
Example of a Bakery

To bake today:
- Croissants
- Pie
- French Bread
- Whole-wheat Muffins Peppernuts
Example of a Bakery

To bake today:

- 1 whole-wheat muffins
- 2 pie
- Peppernuts
- French bread

Time to bake:

- 8
- 8
- 8
Example of a Bakery

To bake today:
- French Bread
- Whole-wheat Muffins
- Peppernuts
- Croissants
- Pie
- French Bread
- Whole-wheat Muffins
- Peppernuts
Example of a Bakery

To bake today:

- 1: Whole-wheat
- 2: Muffins
- 2: Peppernuts
- 6: French Bread
Example of a Bakery

To bake today:
- Muffins (4 units)
- Peppernuts (2 units)
- French Bread (2 units)
- Whole-wheat Muffins (1 unit)
- Pie (6 units)
- Croissants (2 units)
- French Bread (8 units)
- Time to bake: 8 units

Fine-Grained Parameterized Complexity of Scheduling and Sequencing Problems
Example of a Bakery

To bake today:

- Wh
- Croiss
- Pie
- French Bread
- Whole-wheat Muffins
- Peppernuts

Time to bake:

1. 8
2. 8
3. 8

Fine-Grained Parameterized Complexity of Scheduling and Sequencing Problems
Example of a Bakery

To bake today:

- Wh
- Croissant
- Pie
- French Bread
- Whole-wheat Muffins
- Peppernuts

Time to bake:

1. 8
2. 8
3. 8
4. 6
Example of a Bakery

Fine-Grained Parameterized Complexity of Scheduling and Sequencing Problems
Example of a Bakery

Fine-Grained Parameterized Complexity of Scheduling and Sequencing Problems
Example of a Bakery

Time to bake:

- 1, 2, 4
- 2, 6
- 4, 4

- Baked goods: 8, 8, 8
**Bin Packing**

**Given:**
- $n$ items
- $w_j \in \mathbb{N}$ integer weight of item $j$
- $m$ bins with capacity $c$

**Question:** Can the items be distributed over the bins?

**Bakery example:**
- 8 items
- $w = (1, 2, 2, 4, 4, 4, 6)$
- 3 bins with capacity 8
How to solve **BIN PACKING**?

Try all possibilities!

Each item: $m$ possibilities.

Total of $m \cdot m \cdot \cdots \cdot m = m^n$ possibilities!

⇒ Check for each if solution!

Example: if $m = 3$, then $3^n$ possibilities.
How to solve **Bin Packing**?

Try all possibilities!

Each item: \( m \) possibilities.

Total of \( m \cdot m \cdot \cdots \cdot m = m^n \) possiblities!

⇒ Check for each if solution!

Example: if \( m = 3 \), then \( 3^n \) possiblities.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( 3^n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>4</td>
<td>81</td>
</tr>
<tr>
<td>5</td>
<td>243</td>
</tr>
<tr>
<td>6</td>
<td>729</td>
</tr>
<tr>
<td>7</td>
<td>2,187</td>
</tr>
<tr>
<td>8</td>
<td>6,561</td>
</tr>
<tr>
<td>9</td>
<td>19,683</td>
</tr>
<tr>
<td>10</td>
<td>59,049</td>
</tr>
<tr>
<td>11</td>
<td>177,147</td>
</tr>
<tr>
<td>12</td>
<td>531,441</td>
</tr>
<tr>
<td>13</td>
<td>1,594,323</td>
</tr>
<tr>
<td>14</td>
<td>4,782,969</td>
</tr>
<tr>
<td>15</td>
<td>14,348,907</td>
</tr>
<tr>
<td>16</td>
<td>43,046,721</td>
</tr>
<tr>
<td>17</td>
<td>129,140,163</td>
</tr>
<tr>
<td>18</td>
<td>387,420,489</td>
</tr>
<tr>
<td>19</td>
<td>1,162,261,467</td>
</tr>
<tr>
<td>20</td>
<td>3,486,784,401</td>
</tr>
</tbody>
</table>

### Approximate Time

- \( \approx \) minutes
- \( \approx \) hours
- > day
How to solve **Bin Packing**?

**Try all possibilities!**

- **Enumeration**
  - \( m^n \cdot \text{poly}(n) \)

**Use known techniques**

- **Sort & Search** [Lenté et al., 2013]
  - \( m^{n/2} \cdot \left( \frac{n}{2} \right)^\mathcal{O}(m) \)

- **Dynamic Programming**
  - \( \mathcal{O}(n^2 \cdot \text{capacity}^m) \)

- **Fast Subset Convolution** [Björklund et al., 2007]
  - \( 2^n \cdot \text{poly}(n) \)

---

Fine-Grained Parameterized Complexity of Scheduling and Sequencing Problems
# How to solve **Bin Packing**?

- **Try all possibilities!**
- **Use known techniques**

<table>
<thead>
<tr>
<th>$n$</th>
<th>$3^n$</th>
<th>$2^n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>81</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>243</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>729</td>
<td>64</td>
</tr>
<tr>
<td>7</td>
<td>2.187</td>
<td>128</td>
</tr>
<tr>
<td>8</td>
<td>6.561</td>
<td>256</td>
</tr>
<tr>
<td>9</td>
<td>19.683</td>
<td>512</td>
</tr>
<tr>
<td>10</td>
<td>59.049</td>
<td>1.024</td>
</tr>
<tr>
<td>11</td>
<td>177.147</td>
<td>2.048</td>
</tr>
<tr>
<td>12</td>
<td>531.441</td>
<td>4.096</td>
</tr>
<tr>
<td>13</td>
<td>1.594.323</td>
<td>8.192</td>
</tr>
<tr>
<td>14</td>
<td>4.782.969</td>
<td>16.384</td>
</tr>
<tr>
<td>15</td>
<td>14.348.907</td>
<td>32.768</td>
</tr>
<tr>
<td>16</td>
<td>43.046.721</td>
<td>65.536</td>
</tr>
<tr>
<td>17</td>
<td>129.140.163</td>
<td>131.072</td>
</tr>
<tr>
<td>18</td>
<td>387.420.489</td>
<td>262.144</td>
</tr>
<tr>
<td>19</td>
<td>1.162.261.467</td>
<td>524.288</td>
</tr>
<tr>
<td>20</td>
<td>3.486.784.401</td>
<td>1.048.576</td>
</tr>
</tbody>
</table>

$\approx$ minute
How to solve **BIN PACKING**?

- Try all possibilities!
- Use known techniques

<table>
<thead>
<tr>
<th>n</th>
<th>$3^n$</th>
<th>$2^n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>81</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>243</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>729</td>
<td>64</td>
</tr>
<tr>
<td>7</td>
<td>2,187</td>
<td>128</td>
</tr>
<tr>
<td>8</td>
<td>6,561</td>
<td>256</td>
</tr>
<tr>
<td>9</td>
<td>19,683</td>
<td>512</td>
</tr>
<tr>
<td>10</td>
<td>59,049</td>
<td>1,024</td>
</tr>
<tr>
<td>11</td>
<td>177,147</td>
<td>2,048</td>
</tr>
<tr>
<td>12</td>
<td>531,441</td>
<td>4,096</td>
</tr>
<tr>
<td>13</td>
<td>1,594,323</td>
<td>8,192</td>
</tr>
<tr>
<td>14</td>
<td>4,782,969</td>
<td>16,384</td>
</tr>
<tr>
<td>15</td>
<td>14,348,907</td>
<td>32,768</td>
</tr>
<tr>
<td>16</td>
<td>43,046,721</td>
<td>65,536</td>
</tr>
<tr>
<td>17</td>
<td>129,140,163</td>
<td>131,072</td>
</tr>
<tr>
<td>18</td>
<td>387,420,489</td>
<td>262,144</td>
</tr>
<tr>
<td>19</td>
<td>1,162,261,467</td>
<td>524,288</td>
</tr>
<tr>
<td>20</td>
<td>3,486,784,401</td>
<td>1,048,576</td>
</tr>
<tr>
<td>40</td>
<td>1,099,511,627,776</td>
<td>≈ minute</td>
</tr>
</tbody>
</table>

Fine-Grained Parameterized Complexity of Scheduling and Sequencing Problems
Can we do better?

NP-hard (even for $m = 2$) \( \Rightarrow \) no \( poly(n) \) time algorithm assuming \( P \neq NP \)

“I can’t find an efficient algorithm, but neither can all these famous people.”

[Garey and Johnson 1979, updated version from TU Vienna, https://www.ac.tuwien.ac.at/people/szeider/cartoon/]
Can we do a bit better?

Problems that can be solved in $O((2 - \varepsilon)^n)$ time for $\varepsilon > 0$:

- **Hamiltonian Cycle** in undirected graphs [Björklund 2014]
- **Dominating Set** [Fomin, Grandoni, Kratsch, 2009]
- **Connected Dominating Set** [Fomin, Grandoni, Kratsch, 2008]
- **Multicut** [Lokshtanov, Saurabh, Suchý, 2014]
- **Pathwidth** [Suchan, Villanger, 2009]
- ...

? **Bin Packing** ?
Can we do a bit better?

Problems that *can* be solved in $O((2 - \varepsilon)^n)$ time for a $\varepsilon > 0$:

- **HAMILTONIAN CYCLE** in undirected graphs [Björklund, 2014]
- **DOMINATING SET** [Fomin, Grandoni, Kratsch, 2009]
- **CONNECTED DOMINATING SET** [Fomin, Grandoni, Kratsch, 2008]
- **MULTICUT** [Lokshtanov, Saurabh, Suchý, 2014]
- **PATHWIDTH** [Suchan, Villanger, 2009]
- ...  

**!! BIN PACKING** with constant number of bins [Chapter 4]
Can we do better in special cases?

Find parameter such that when small ⇒ faster algorithm

**Example:** **Bin Packing**, with parameter *capacity*

Dynamic Programming:

$$O(n^2 \cdot \text{capacity}^m) = \text{poly}(n) \cdot f(\text{capacity})$$

If baking time is ‘short’, ⇒ *capacity* is small!

⇒ poly(n) \cdot (\text{capacity})^3
This dissertation

Part 1: Introduction

Part 2: Scheduling Problems

- Chapter 4: **Bin Packing** with constant number of bins
- Chapter 5: $P|prec, p_j = 1|C_{\text{max}}$
- Chapter 6: **Partial Scheduling** (parameterized by the number of jobs to process)

Part 3: Sequencing Problems

- Chapter 7: **Connected Flow** (parameterized by vertex cover, treewidth, and demand vertices)
- Chapter 8: **Hamiltonian Cycle** (parameterized by treedepth)
A Short Introduction into Fine-Grained and Parameterized Complexity

C.M.F. Swennenhuis

Department of Mathematics and Computer Science, Eindhoven University of Technology