A Short Introduction into Fine-Grained and Parameterized Complexity

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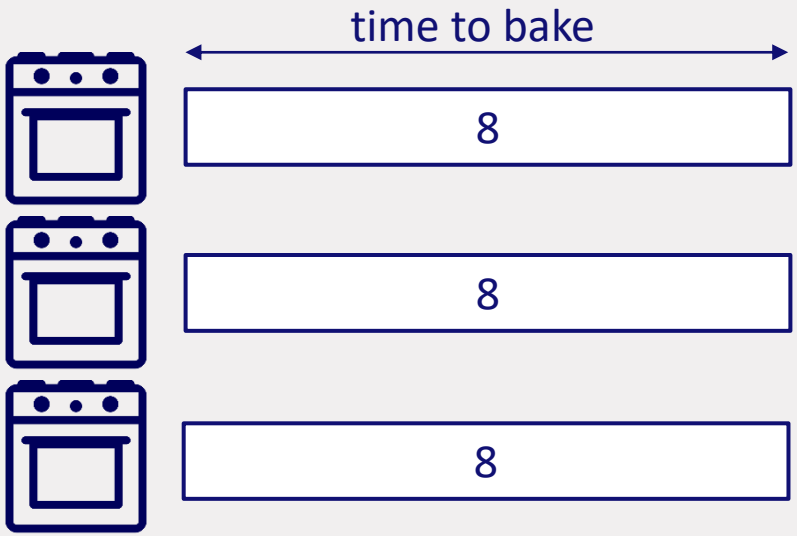
Example of a Bakery

To bake today:

- ❑ White bread
- ❑ Whole-wheat
- ❑ Croissants
- ❑ Muffins
- ❑ Pie
- ❑ Peppernuts
- ❑ French Bread

Example of a Bakery



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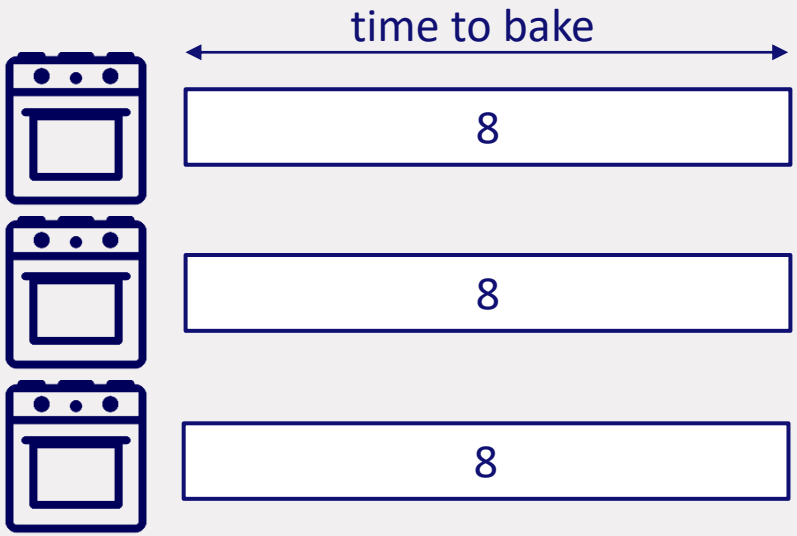
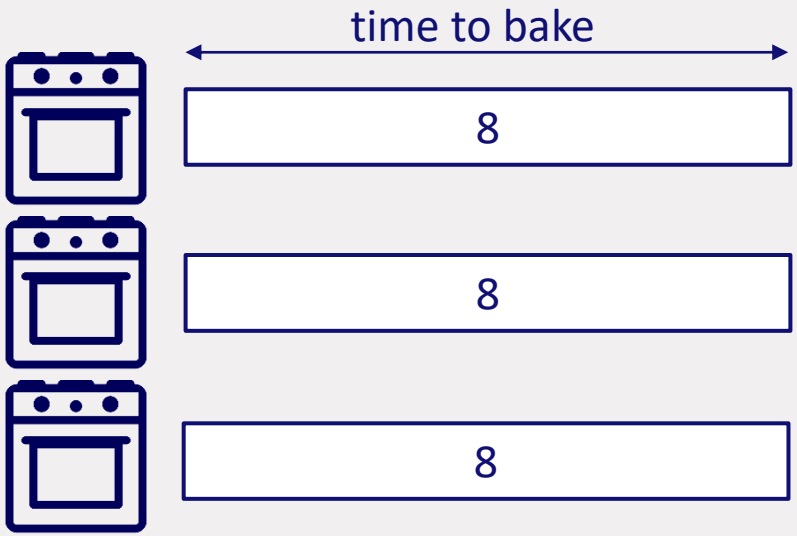


To bake today:

- ❑ 1
- ❑ Whole-wheat
- ❑ Croissants
- ❑ Muffins
- ❑ Pie
- ❑ Peppernuts
- ❑ French Bread

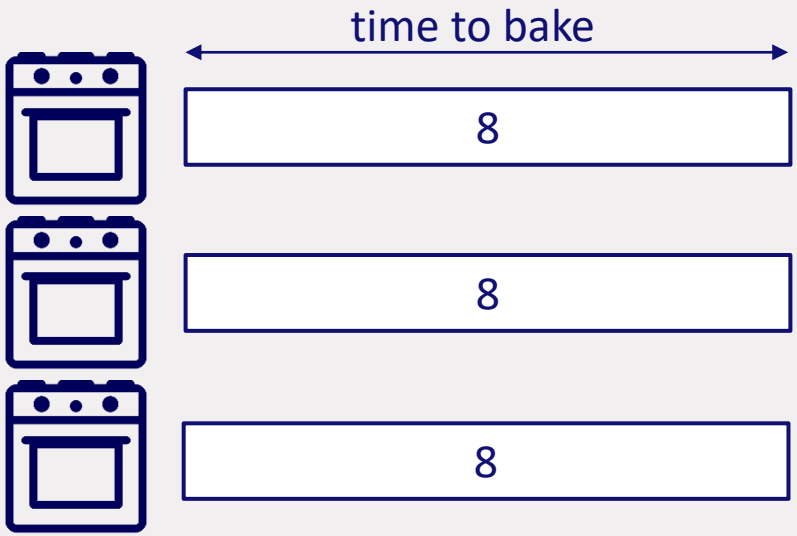
Example of a Bakery



To bake today:

- ❑ 1
- ❑ 2
- ❑ Pie
- ❑ French Bread
- ❑ Whole-wheat
- ❑ Muffins
- ❑ Peppernuts

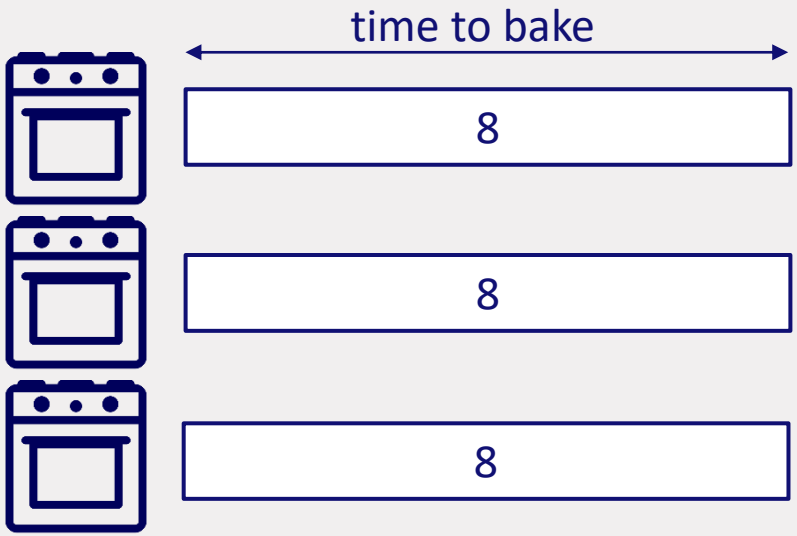
Example of a Bakery



To bake today:

- ❑ 1
- ❑ 2
- ❑ 2
- ❑ French Bread
- ❑ Whole-wheat
- ❑ Muffins
- ❑ Peppernuts

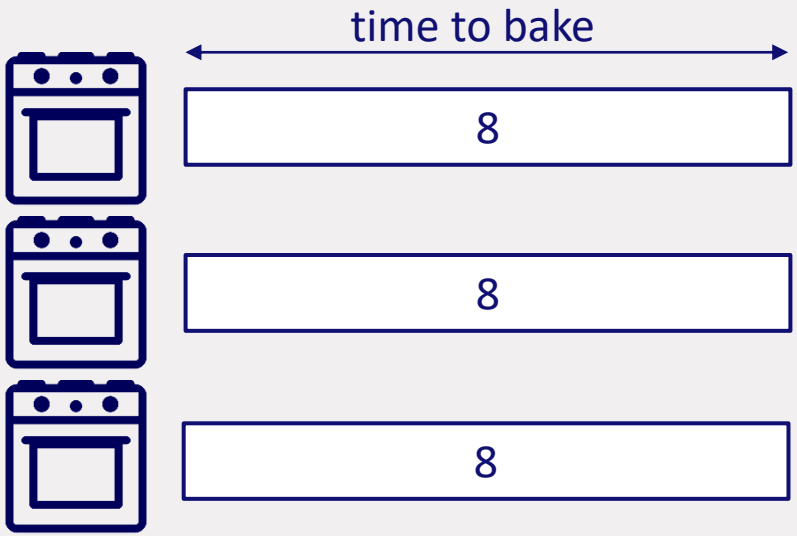
Example of a Bakery



To bake today:

- ❑ 1 ❑ Whole-wheat
- ❑ 2 ❑ Muffins
- ❑ 2 ❑ Peppernuts
- ❑ 6

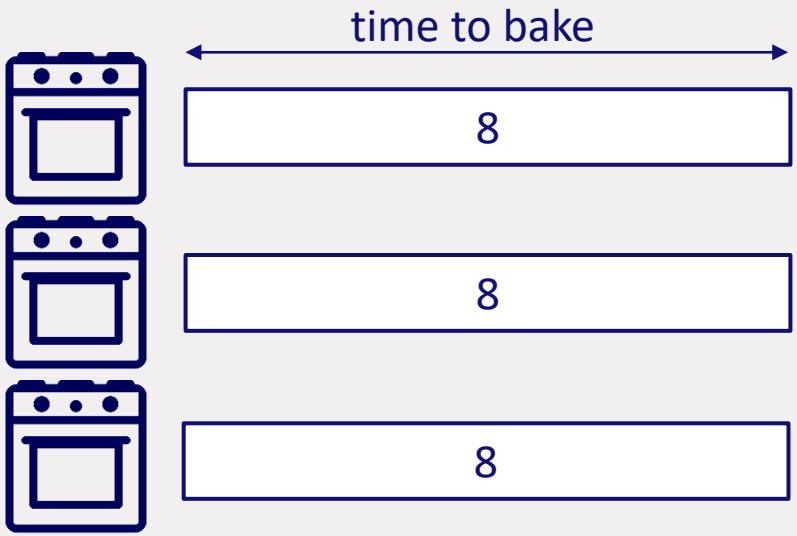
Example of a Bakery



To bake today:

- ❑ 1
- ❑ 4
- ❑ 2
- ❑ Muffins
- ❑ 2
- ❑ Peppernuts
- ❑ 6

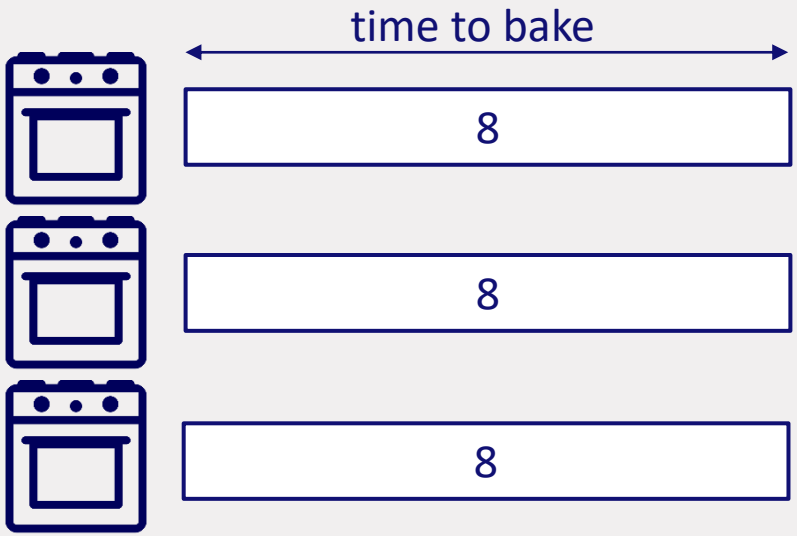
Example of a Bakery



To bake today:

- ❑ 1
- ❑ 2
- ❑ 2
- ❑ 6
- ❑ 4
- ❑ 4
- ❑ Peppernuts

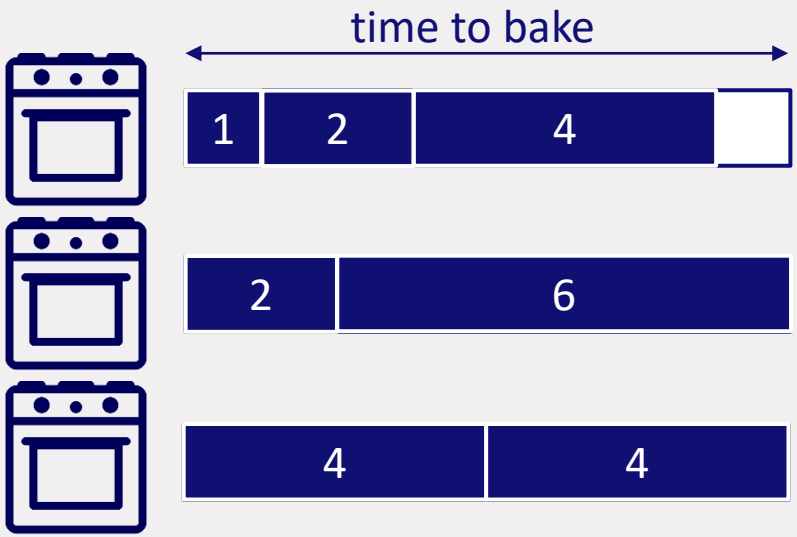
Example of a Bakery



To bake today:

- ❑ 1
- ❑ 2
- ❑ 2
- ❑ 6
- ❑ 4
- ❑ 4
- ❑ 4

Example of a Bakery



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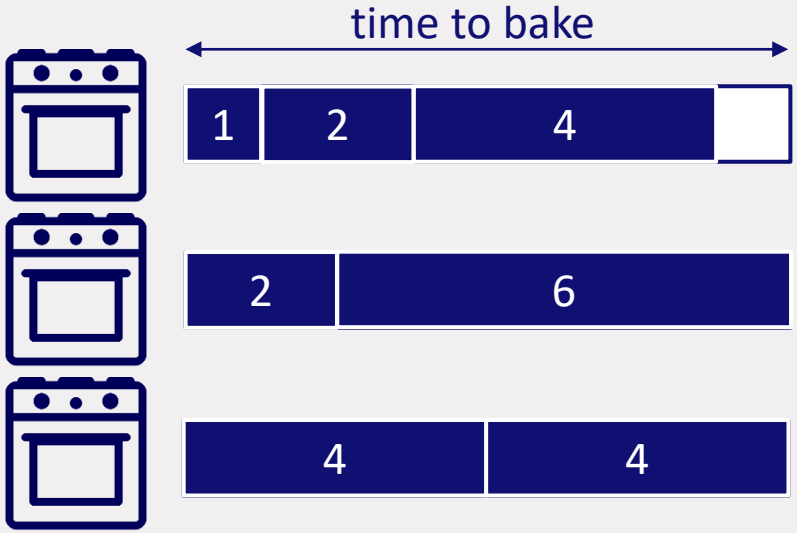
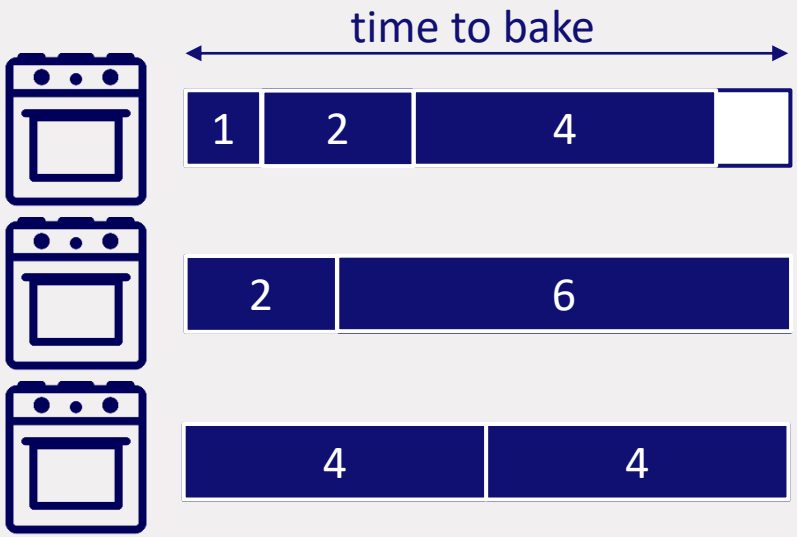
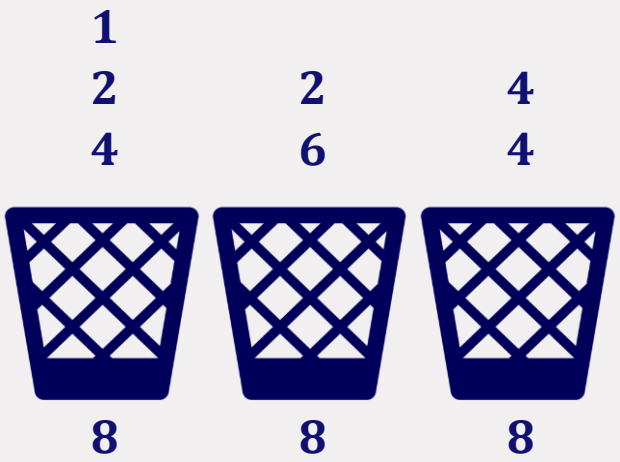


1 2 2 4 4 4 6

8 8 8

Example of a Bakery





Bin Packing

Given:

- n items
- $w_j \in \mathbb{N}$ integer weight of item j
- m bins with capacity c

Bakery example:

- 8 items
- $w = (1, 2, 2, 4, 4, 4, 6)$
- 3 bins with capacity 8

Question: Can the items be distributed over the bins?

How to solve BIN PACKING?

Try all possibilities!

Each item: m possibilities.

Total of $m \cdot m \cdot \cdots \cdot m = m^n$ possiblities!

<u>⇒ Check for each if solution!</u>

Example: if $m = 3$, then 3^n possiblities.

How to solve BIN PACKING?

Try all possibilities!

Each item: m possibilities.

Total of $m \cdot m \cdot \cdots \cdot m = m^n$ possiblities!

<u>⇒ Check for each if solution!</u>

Example: if $m = 3$, then 3^n possiblities.

n	3^n	
1	3	
2	9	
3	27	
4	81	
5	243	
6	729	
7	2.187	
8	6.561	
9	19.683	
10	59.049	
11	177.147	
12	531.441	
13	1.594.323	
14	4.782.969	≈ minutes
15	14.348.907	
16	43.046.721	
17	129.140.163	
18	387.420.489	≈ hours
19	1.162.261.467	
20	3.486.784.401	> day

How to solve BIN PACKING?

Try all possibilities!

Use known techniques

- Enumeration
  - $m^n \cdot poly(n)$
- Sort & Search [Lenté et al., 2013]
  - $m^{n/2} \cdot \left(\frac{n}{2}\right)^{\mathcal{O}(m)}$
- Dynamic Programming
  - $\mathcal{O}(n^2 \cdot capacity^m)$
- Fast Subset Convolution [Björklund et al., 2007]
  - $2^n \cdot poly(n)$

How to solve BIN PACKING?

- Try all possibilities!
- Use known techniques

n	3^n	2^n
1	3	2
2	9	4
3	27	8
4	81	16
5	243	32
6	729	64
7	2.187	128
8	6.561	256
9	19.683	512
10	59.049	1.024
11	177.147	2.048
12	531.441	4.096
13	1.594.323	8.192
14	4.782.969	16.384
15	14.348.907	32.768
16	43.046.721	65.536
17	129.140.163	131.072
18	387.420.489	262.144
19	1.162.261.467	524.288
20	3.486.784.401	1.048.576

≈ minute

How to solve BIN PACKING?

- Try all possibilities!
- Use known techniques

n	3^n	2^n
1	3	2
2	9	4
3	27	8
4	81	16
5	243	32
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9	19.683	512
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15	14.348.907	32.768
16	43.046.721	65.536
17	129.140.163	131.072
18	387.420.489	262.144
19	1.162.261.467	524.288
20	3.486.784.401	1.048.576
40		1.099.511.627.776

≈ minute

Can we do better?

NP-hard (even for $m = 2$) ⇒ no $poly(n)$ time algorithm assuming $\mathbf{P} \neq \mathbf{NP}$

"I can't find an efficient algorithm, but neither can all these famous people."

[Garey and Johnson 1979, updated version from TU Vienna, https://www.ac.tuwien.ac.at/people/szeider/cartoon/]

Can we do a bit better?

Fine-Grained Complexity

Problems that *can* be solved in $\mathcal{O}\left((2-\varepsilon)^n\right)$ time for a $\varepsilon > 0$:

- ✓ HAMILTONIAN CYCLE in undirected graphs [Björklund 2014]
- ✓ DOMINATING SET [Fomin, Grandoni, Kratsch, 2009]
- ✓ CONNECTED DOMINATING SET [Fomin, Grandoni, Kratsch, 2008]
- ✓ MULTICUT [Lokshtanov, Saurabh, Suchý, 2014]
- ✓ PATHWIDTH [Suchan, Villanger, 2009]
- ✓ …

? BIN PACKING ?

Can we do a bit better?

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- ✓ PATHWIDTH [Suchan, Villanger, 2009]
- ✓ …

!! BIN PACKING with constant number of bins [Chapter 4]

Can we do better in special cases?

Parameterized Complexity

Find parameter such that when *small* ⇒ faster algorithm

Example: BIN PACKING, with parameter $capacity$

Dynamic Programming:

$$\mathcal{O}(n^2 \cdot capacity^m) = poly(n) \cdot f(capacity)$$

If baking time is 'short', ⇒ $capacity$ is *small*!

$$\Rightarrow poly(n) \cdot (capacity)^3$$

This dissertation

Part 1: Introduction

Part 2: Scheduling Problems

- Chapter 4: Bin Packing with constant number of bins
- Chapter 5: $P|prec, p_j = 1|C_{\max}$
- Chapter 6: Partial Scheduling (parameterized by the number of jobs to process)

Part 3: Sequencing Problems

- Chapter 7: Connected Flow (parameterized by vertex cover, treewidth, and demand vertices)
- Chapter 8: Hamiltonian Cycle (parameterized by treedepth)

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