Parallel Machine Scheduling

with a Single Resource per Job

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Motivation: Semi-conductor Industry

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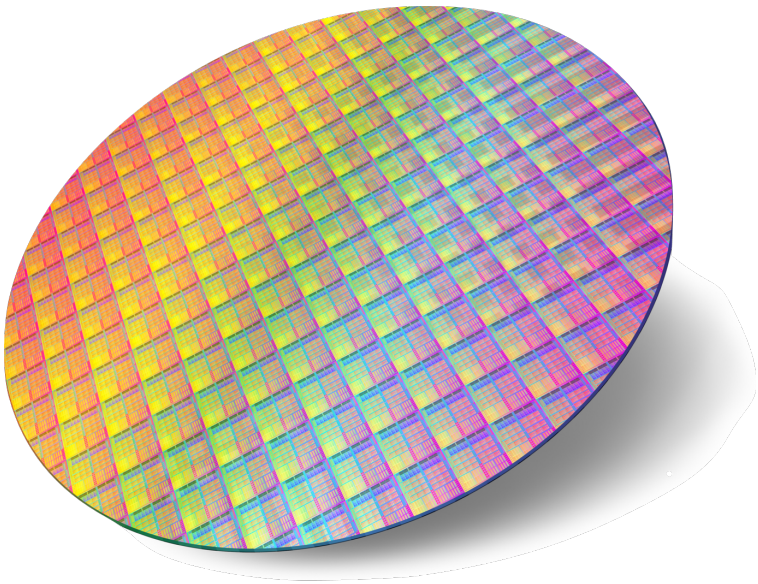

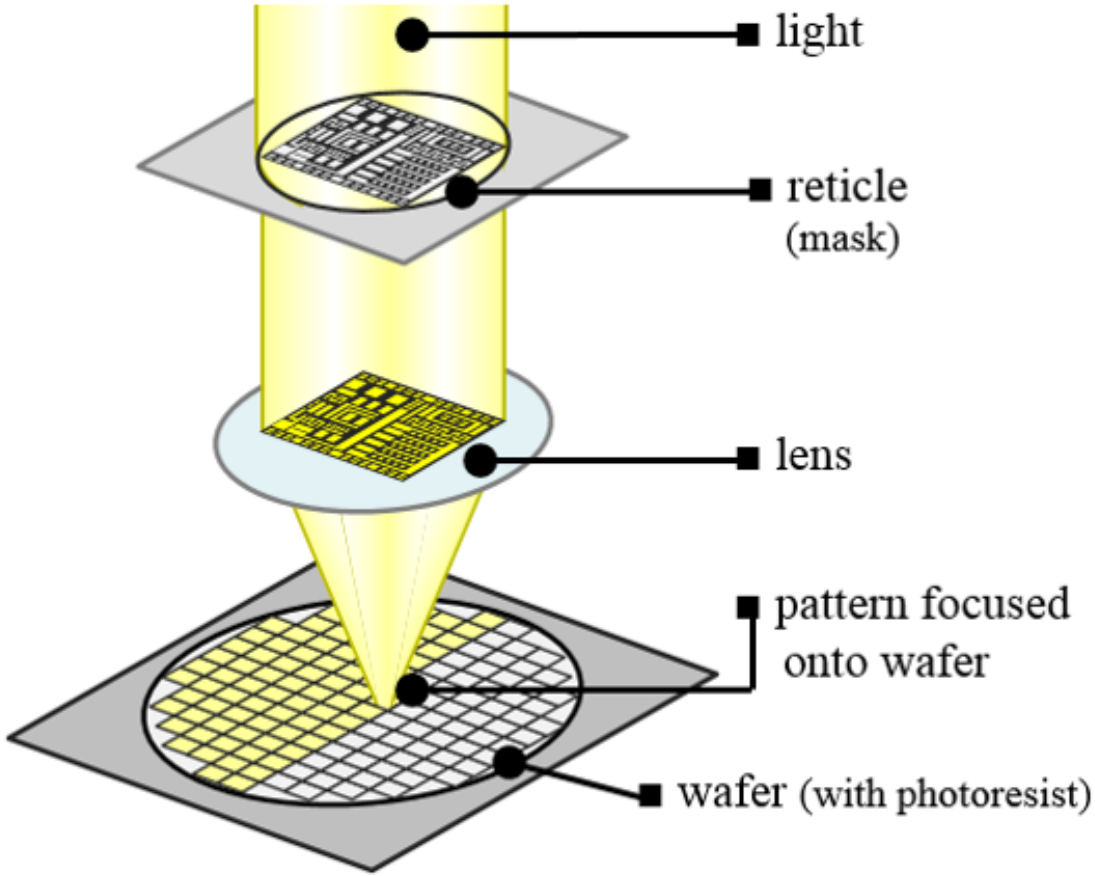

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2. SPT-available Rule
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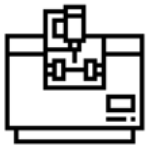

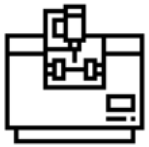

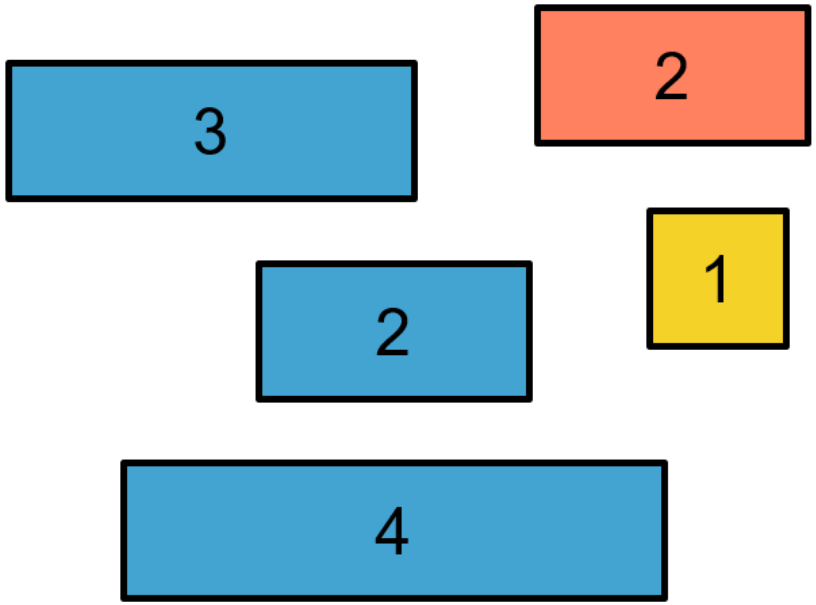

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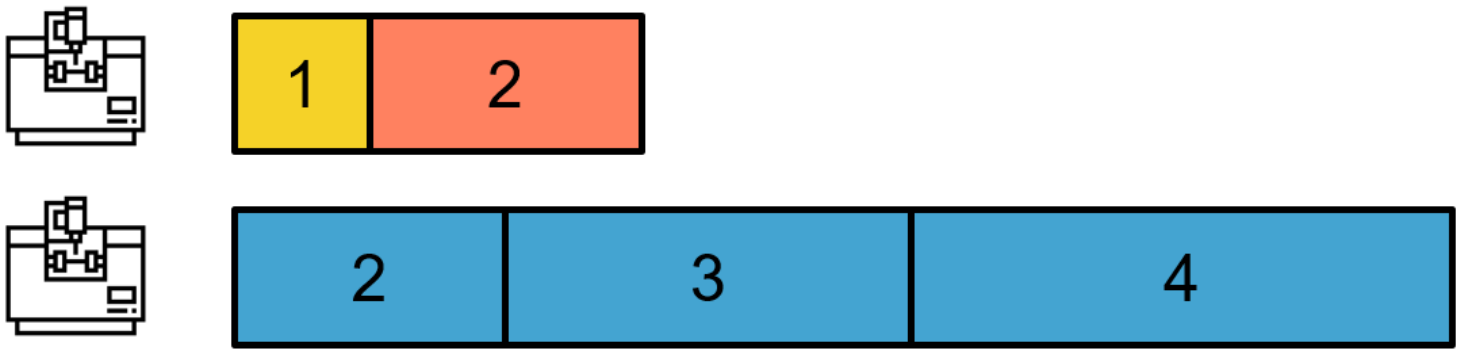

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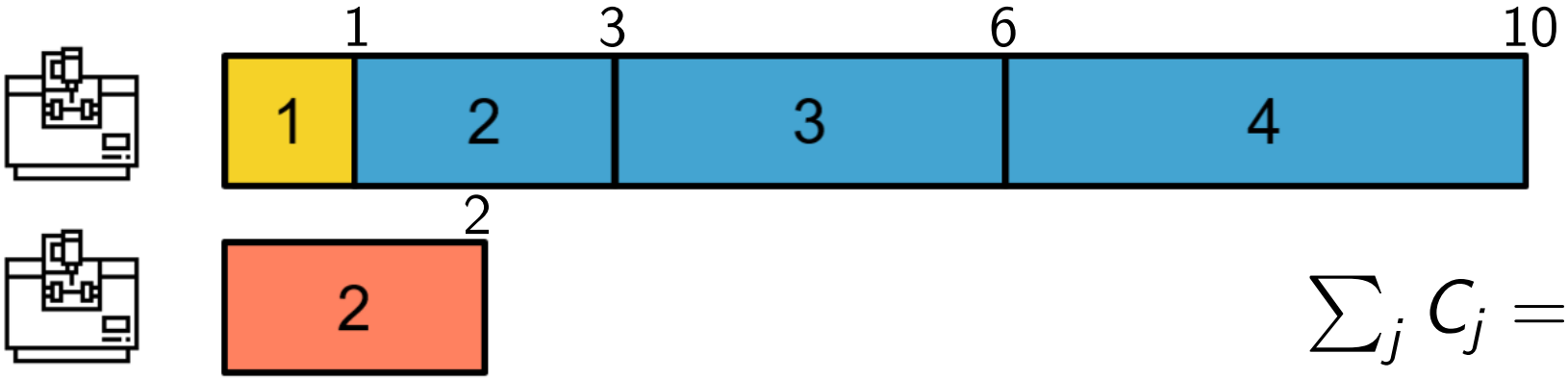
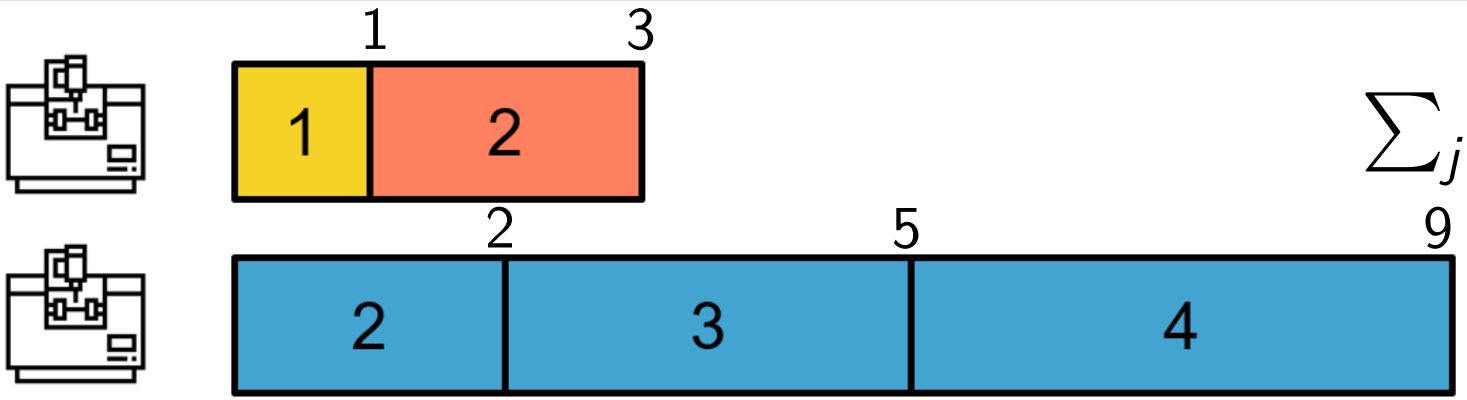


$$\sum_j C_j = 22$$



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Problem Definition

- Input:
  - m machines
  - n jobs with processing times p_j
  - k resources
  - jobs are partitioned into which resource they use
- Schedule feasible
  - When each resource is used at most once at a time
- Objective:
  - Minimize $\sum_j C_j$

We denote[1] this problem by:

$$P|partition|\sum_j C_j$$

[1] Using the notation of [Graham et al., 1979]

Problem Definition

The complexity of $P|partition|\sum_j C_j$ remains open.

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SPT-*available* Rule

Definition (Shortest Processing Time Rule, SPT)

The next job to be scheduled is the job with the shortest processing time.

SPT is exact for $P||\sum_j C_j$ [Smith, 1956]. However, how to deal with the resources?

SPT-*available* Rule

Definition (Shortest Processing Time Rule, SPT)

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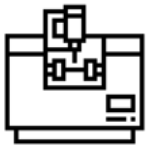
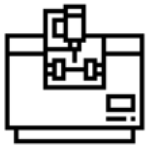
SPT is exact for $P||\sum_j C_j$ [Smith, 1956]. However, how to deal with the resources?

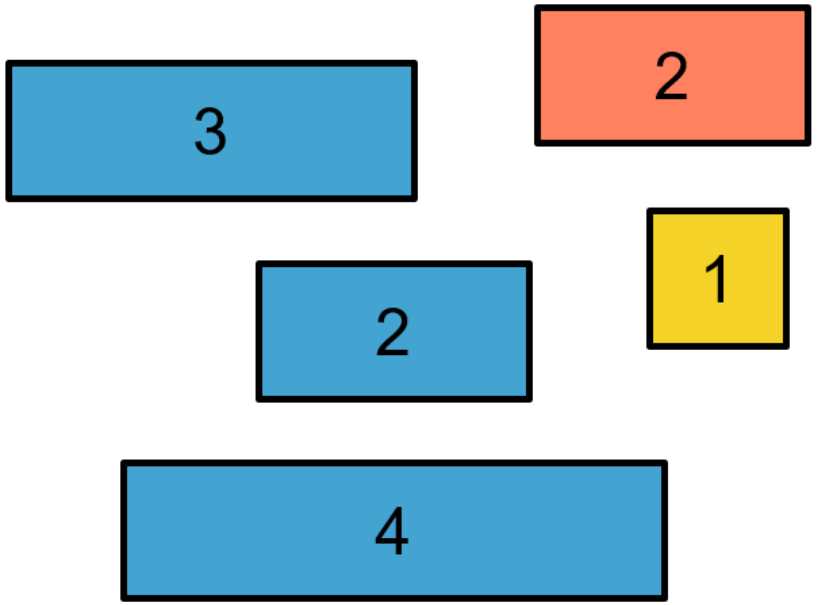
Definition (SPT-*available* Rule)

The next job to be scheduled is the job with the shortest processing time. If its resources is not available, schedule it at the earliest opportunity.

SPT-*available* Rule







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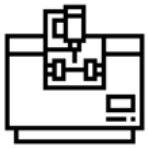

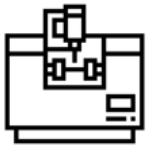

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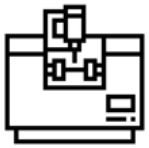

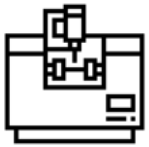

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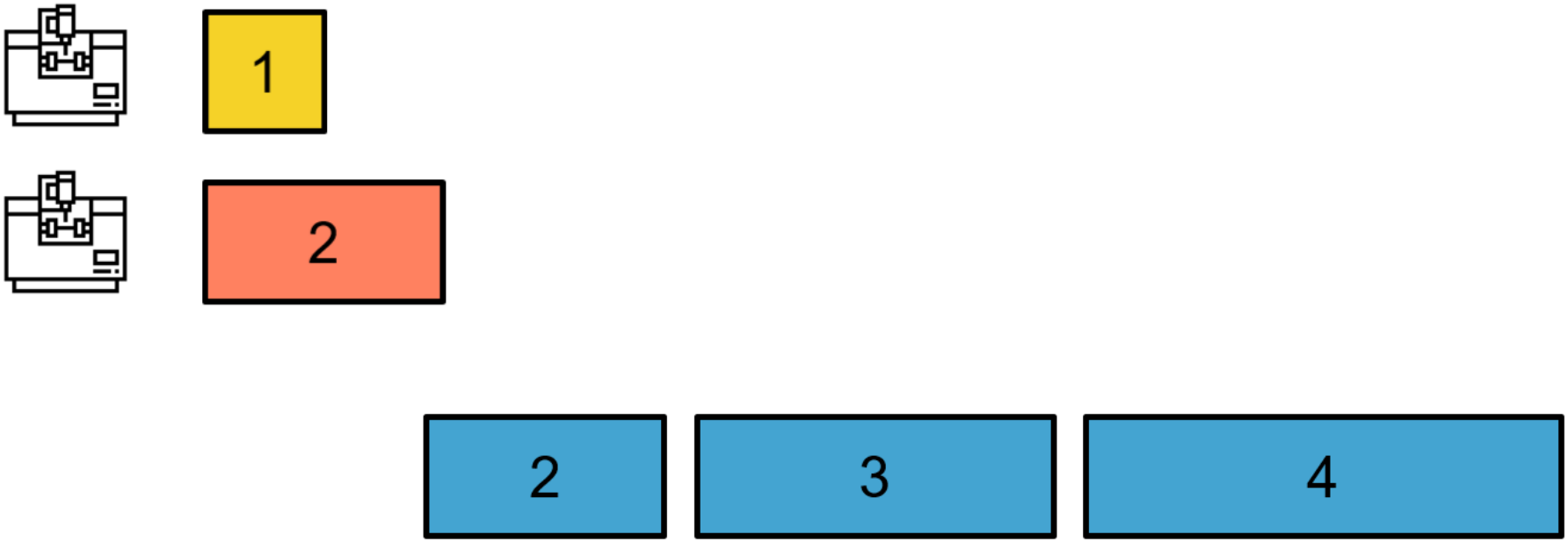

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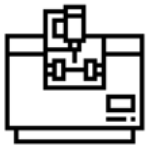

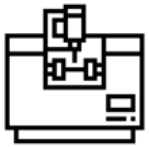

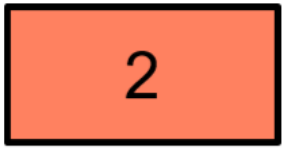

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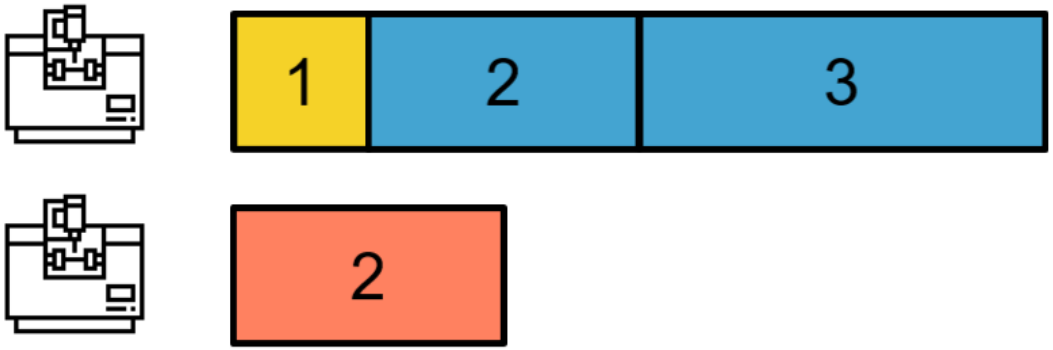

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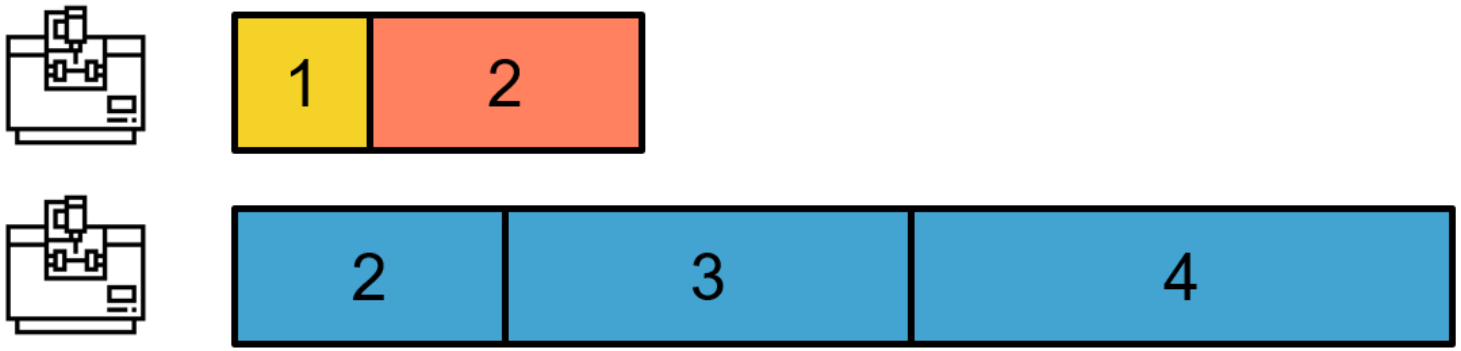

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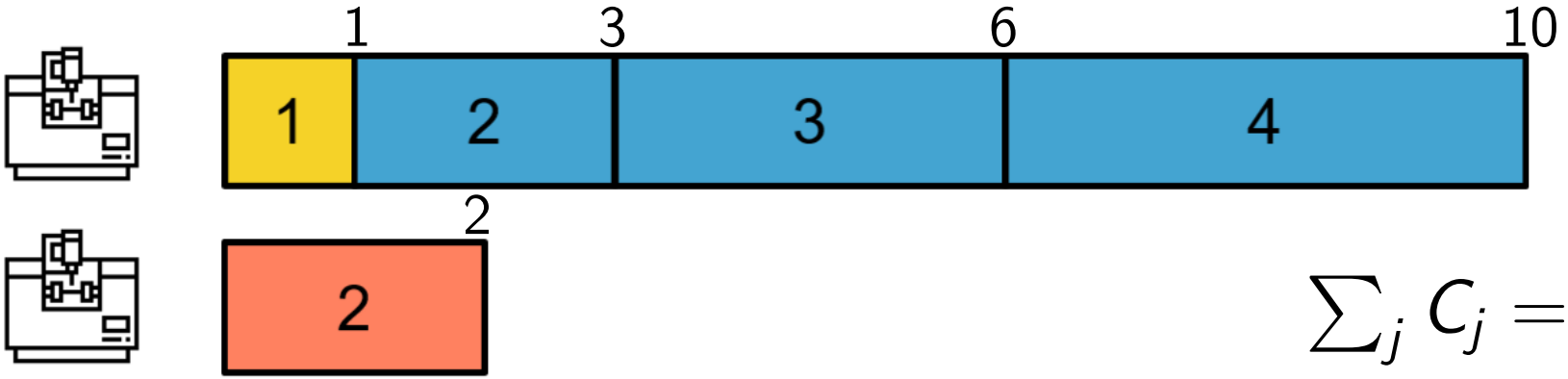

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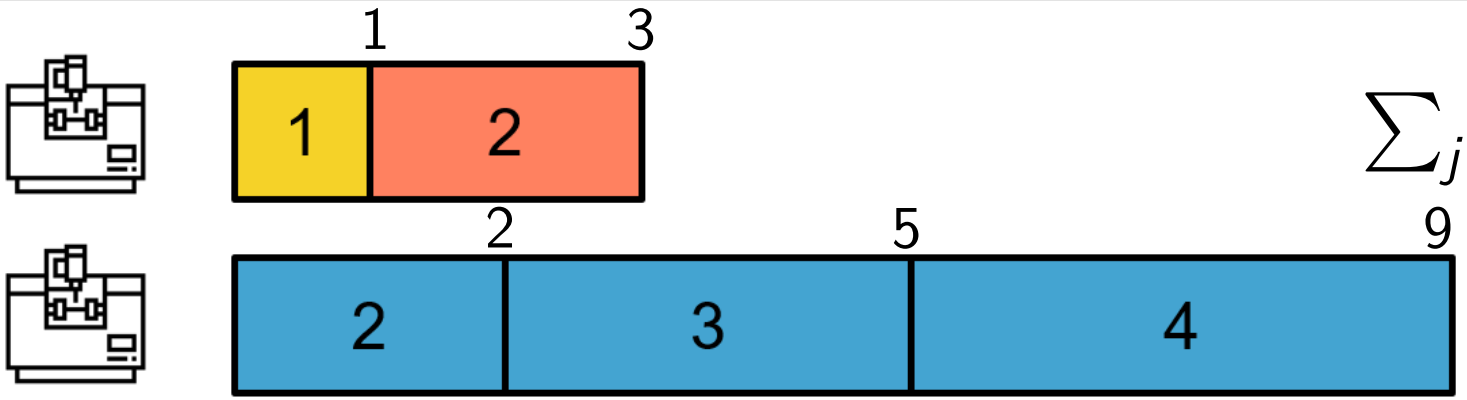

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SPT-*available* Rule

Theorem 2

The SPT-*available* rule gives a $(2 - \frac{1}{m})$-approximation for $P|partition|\sum_j C_j$.

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This bound is not tight. We can construct instances where it gives an α-approximation for $\alpha < \frac{4}{3}$.

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Ordered Resources Property

Theorem 1

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Outline of proof:

$P|partition|\sum_j C_j$

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Outline of proof:

$$P|partition|\sum_j C_j \xrightarrow[\text{preemptions}]{\text{allow}} P|partition,prmp|\sum_j C_j$$

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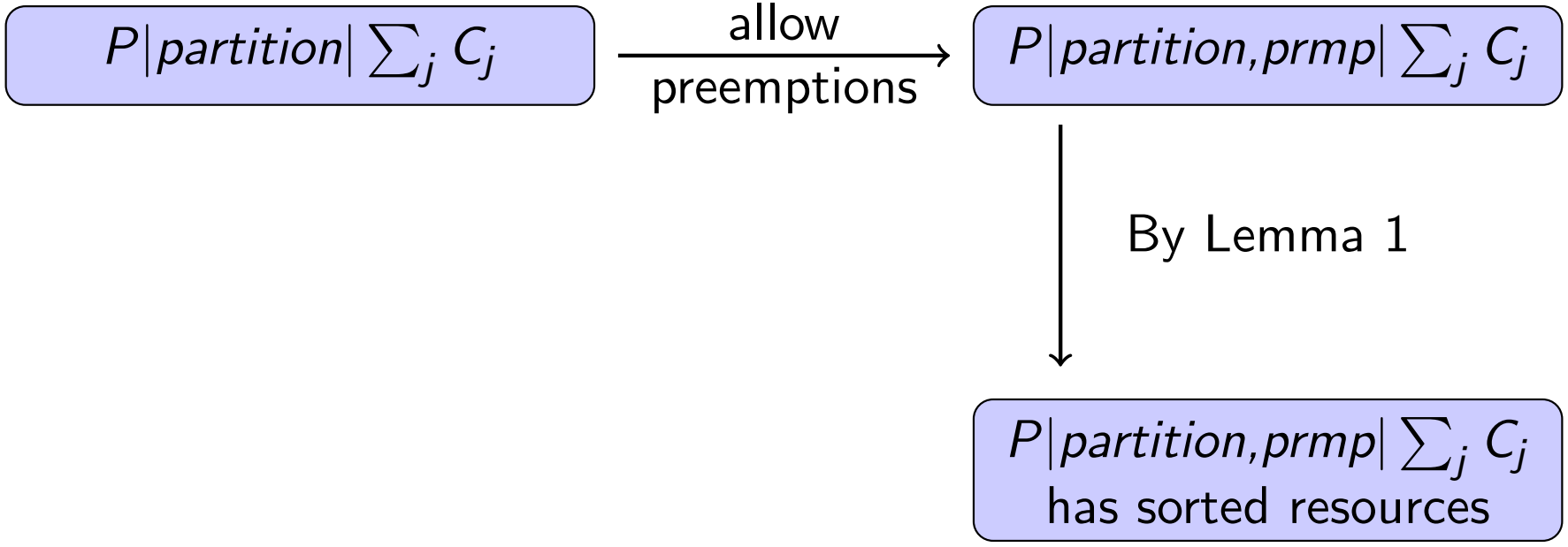

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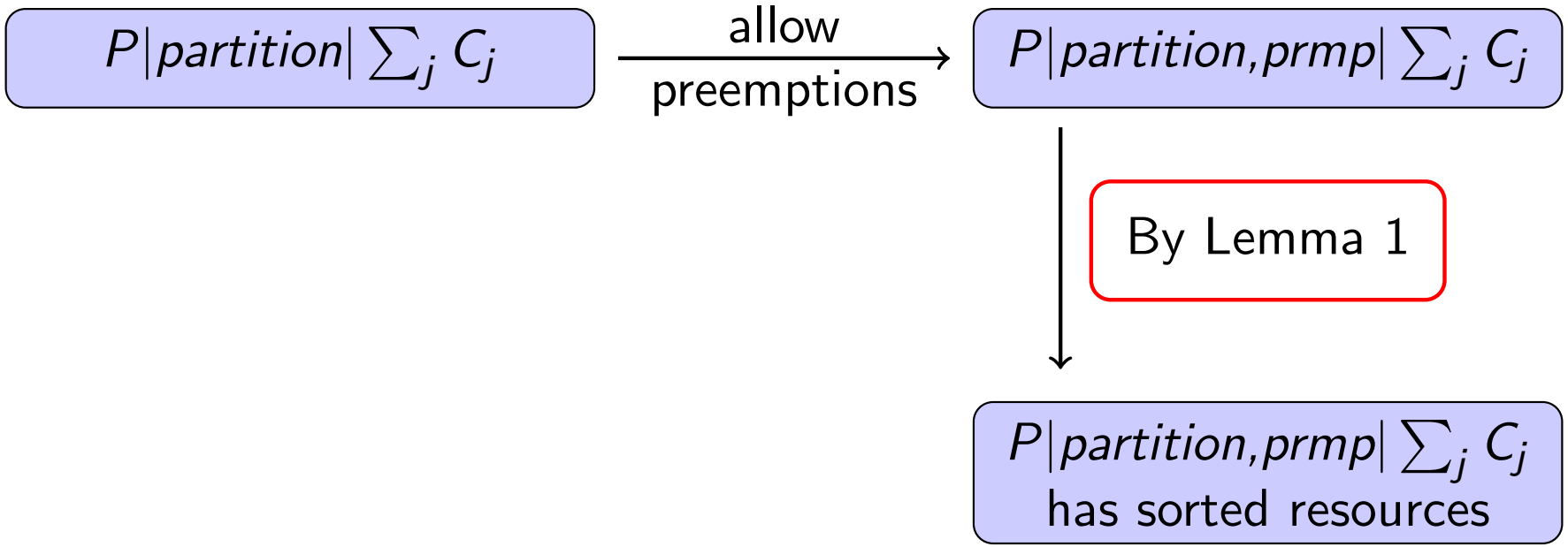

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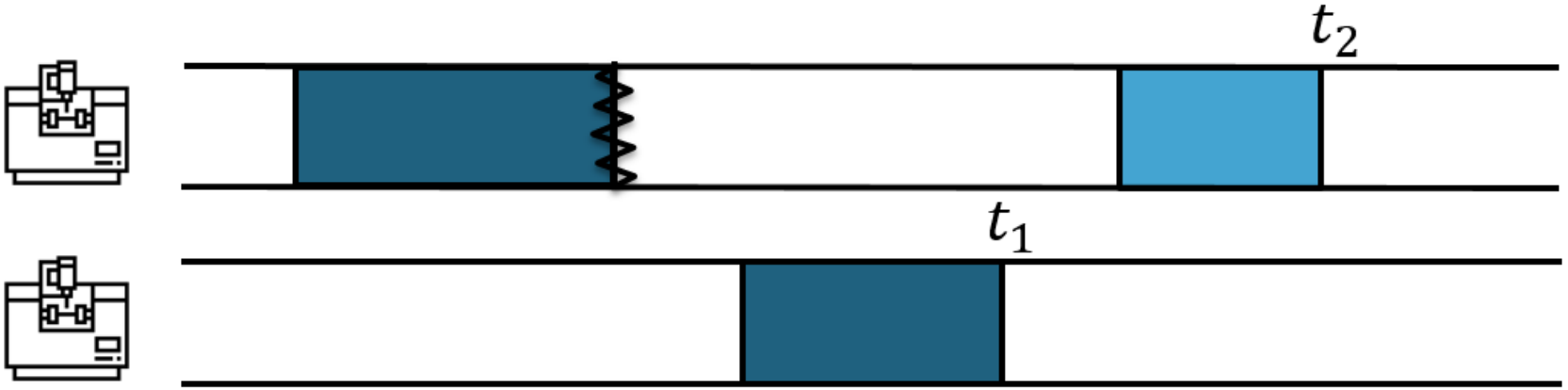

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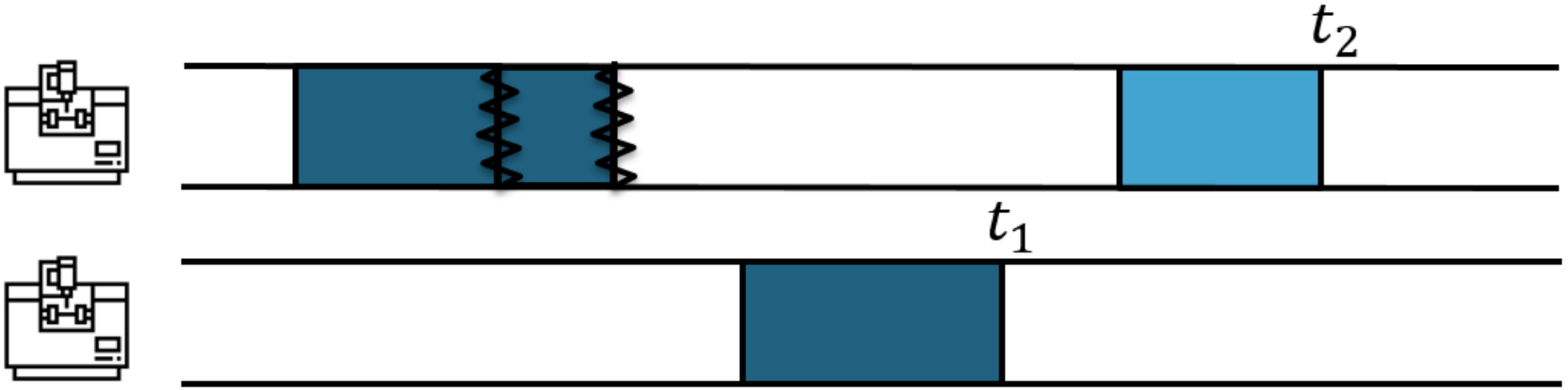

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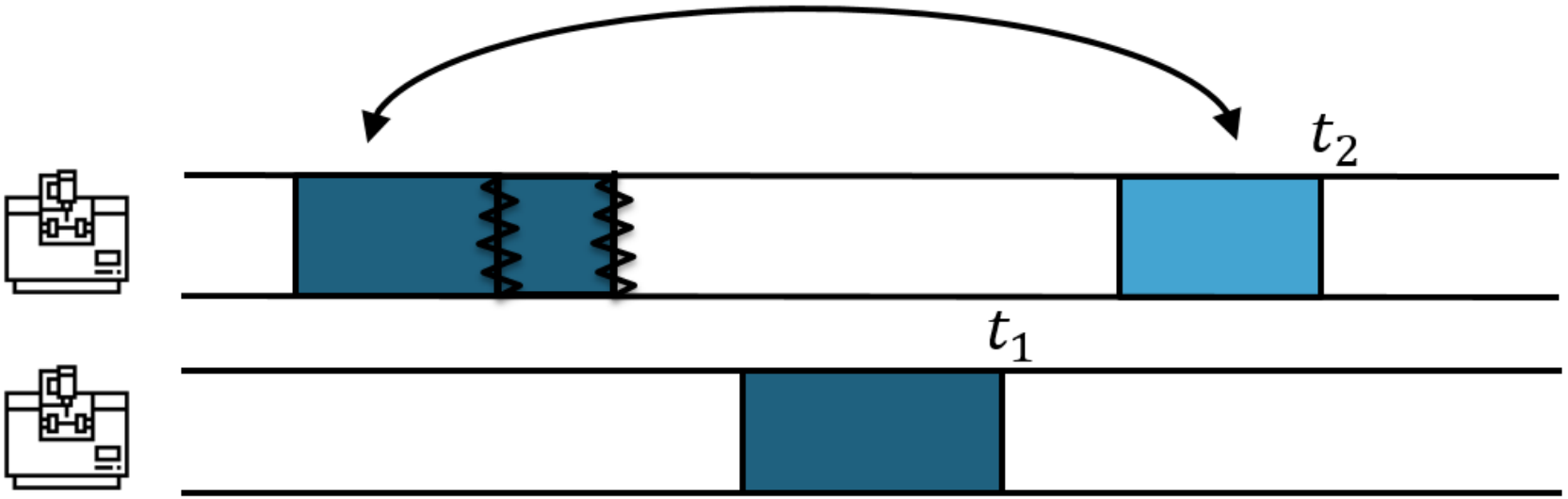

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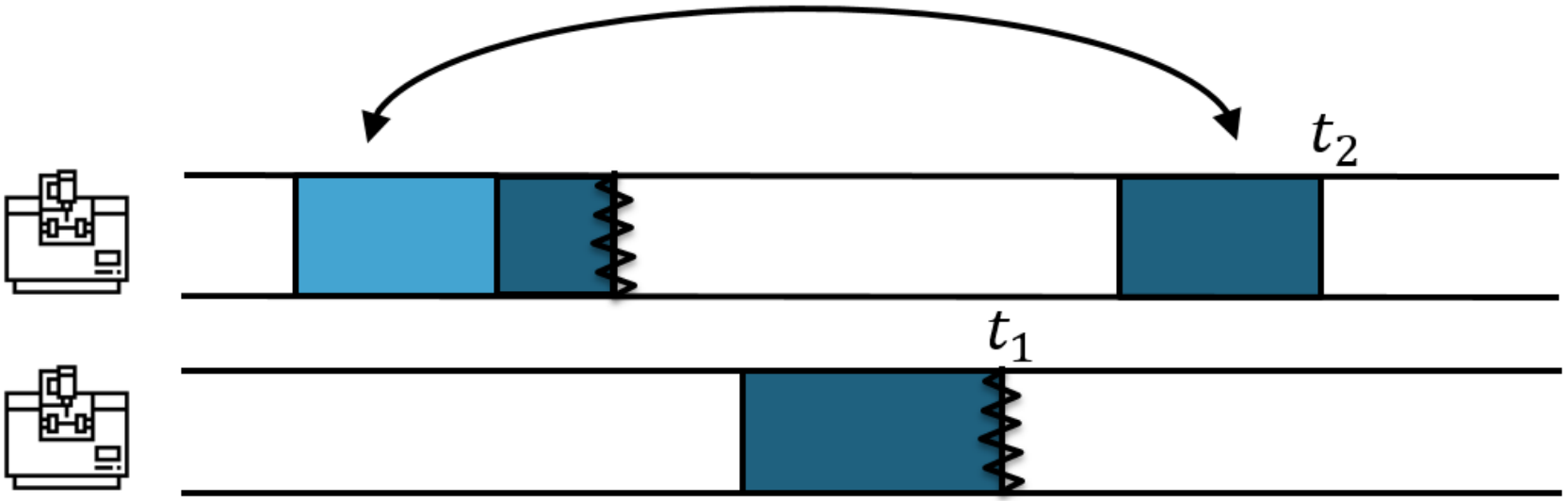

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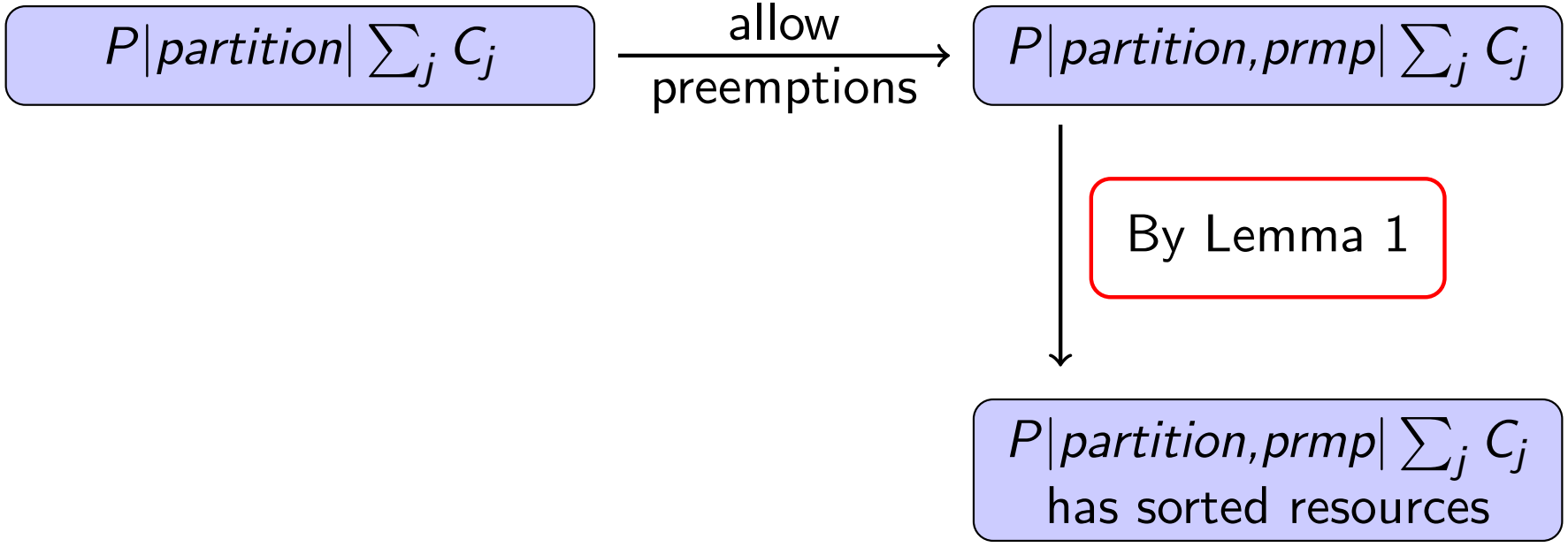

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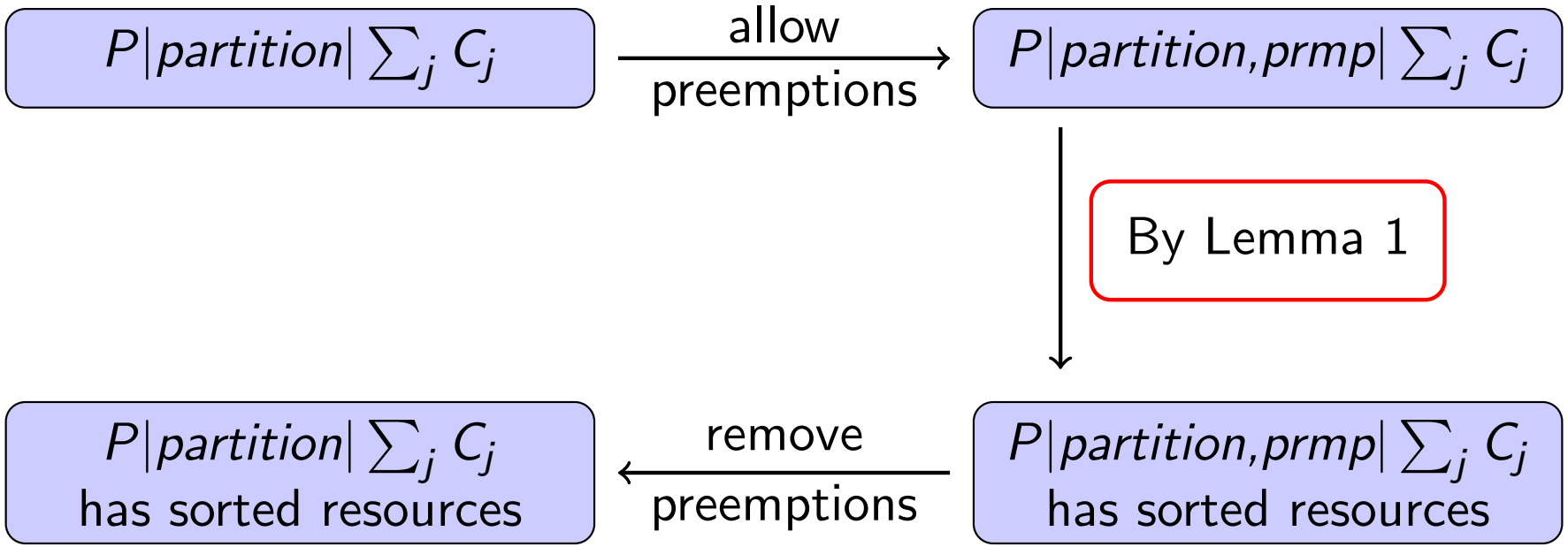

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Practical Solution

In practice, machines are not identical (but unrelated). We proved this problem to be *NP*-hard.

- In practice, every resource (reticle) is used at most 30 times a month
  - Solve easier problem without resources
  - Make schedule feasible
- Results:
  - Gives 6.02% reduction in total completion time
  - Gives 1.97% reduction in machine usage

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Future Work

- Establish the complexity of $P|partition|\sum_j C_j$
- Establish worst-case approximation ratio for SPT-*available* rule.
- Is $P|partition|\sum_j C_j$ fixed-parameter tractable in k (the number of resources)?

References

Graham, R. L., Lawler, E. L., Lenstra, J. K., and Kan, A. R. (1979).
Optimization and approximation in deterministic sequencing and scheduling.
A Survey, Annals of Discrete Mathematics, 5:287–326.

Smith, W. E. (1956).
Various optimizers for single-stage production.
Naval Research Logistics Quarterly, 3(1-2):59–66.

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